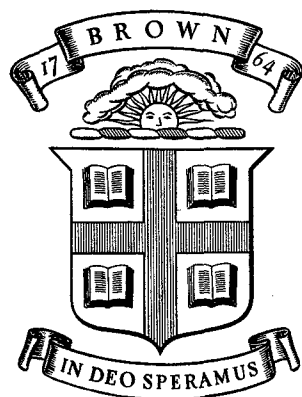


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ENGINEERING MATERIALS RESEARCH LABORATORY

ELEVATED TEMPERATURE CREEP OF
POLYURETHANE UNDER NONLINEAR TORSIONAL
STRESS WITH STEP CHANGES IN TORQUE

J. S. Y. LAI and W. N. FINDLEY

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ELEVATED TEMPERATURE CREEP OF POLYURETHANE UNDER NONLINEAR
TORSIONAL STRESS WITH STEP CHANGES IN TORQUE

by

J. S. Y. Lai^{*} and W. N. Findley^{**}

Summary

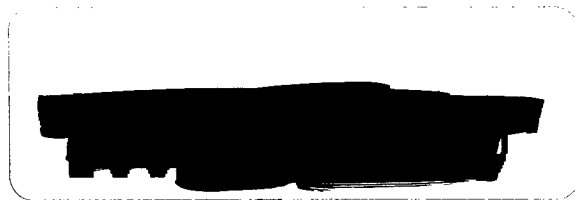
Constant stress experiments, recovery and multi-step creep tests in torsion in the nonlinear range were performed at different temperatures up to 160°F. It was found that the results could be described by a power function of time whose exponent was independent of stress and temperature. Most of the temperature effect and most of the nonlinearity were found in the coefficient of the time-dependent term. The nonlinearity was described by the multiple integral representation, and the changes in stress were adequately predicted by the modified superposition method, except at the highest temperatures.

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* Assistant Professor of Civil Engineering, University of Utah, Salt Lake City, Utah; formerly Research Associate, Brown University, Providence, Rhode Island.

** Professor of Engineering, Brown University, Providence, Rhode Island.



Introduction

It has been found that the time-temperature superposition principle, see for example [1],* satisfactorily accounts for the effect of temperature on creep and stress relaxation behavior of linear viscoelastic polymers. However, there is little experimental information on the effect of temperature on the behavior of nonlinear viscoelastic materials. Previous investigations of nonlinear viscoelasticity were mostly concerned with constant temperature, usually room temperature. Bernstein, Kearsly and Zapas [2] and Lianis [3] considered an extension of thermorheologically-simple linear viscoelasticity to the nonlinear range. Lepper and Heatherington [4] treated the problem from a statistical standpoint. In previous work [5-13] the nonlinear creep and stress relaxation behavior of solid and foam polyurethane and poly (vinyl chloride) were investigated at room temperature under pure tension, pure torsion and combined tension-torsion stress states. It was found that creep strains at room temperature could be separated into time-independent and time-dependent parts. The time-independent part of the strain was weakly nonlinear, that is the deviation of the nonlinear part from the linear part was small in comparison with the linear part; while the time-dependent part of the strain was strongly nonlinear. It was also found in the previous work that retaining the first three orders of stress terms in the multiple integral representation of the constitutive relation gave sufficiently accurate results for the conditions explored.

In the present paper the effect of temperature on creep behavior of a nonlinear viscoelastic material is investigated and a method of analysis is proposed.

*Numbers in brackets identify references listed at the end of this paper.

Material and Specimen

The material used in the experiments was the same as employed in previous work [8,9,10,12,13]. It was a full density solid polyurethane having a specific gravity of 1.25. This material, identified as XR6-77, was produced by CPR Division, the Upjohn Company, and supplied by the Lawrence Radiation Laboratory. The material was prepared from the prepolymer and combined with toluene diisocyanate. The polyester resin was derived from a dicarboxylic acid and a triol. The carbon content was 59 per cent, hydrogen 6.5 per cent, nitrogen 7.5 per cent, chlorine less than 0.3 per cent, ash 0.05 per cent, and the balance oxygen. The material was cured at a temperature of 250°F for two hours.

The specimen was a tubular type with enlarged threaded ends machined from a solid rod. The average outside diameter was 0.9971 in., with a maximum deviation of 0.0003 in., the average wall thickness was 0.05928 in., with a maximum deviation of 0.0005 in., and the gage length was 4.00 in. All experiments were performed on the same specimen.

Experimental Apparatus

The testing machine was designed to test tubular specimens under combined tension, torsion, and internal pressure [14]. It has been used to investigate creep behavior under various loading conditions [5,6,7] and stress relaxation [8,9,10] for plastics.

The loading and measuring devices for torsion are described briefly as follows. The torsion loads were provided by dead weights. The angle of twist was measured with a sensitivity of 3×10^{-6} in./in. by means of a scribed drum attached to one pair of extension rods which were attached to the upper end of the specimen gage length. A pointer attached to another pair of extension rods was attached to the lower end of the gage length. A measuring microscope was

employed to determine the relative angular displacement between pointer and drum.

In order to perform creep tests of polyurethane above room temperature, the following heating devices were added. The specimen was heated in an electric split-tube furnace which was maintained at constant temperature by a C. N. S. Instruments' SirectMark III proportional temperature controller. Due to the poor conductivity of polyurethane, heat was also provided inside the specimen tube in order to achieve a uniform temperature distribution. A copper bar with a diameter slightly less than the inside diameter of the test specimen was inserted inside the specimen and two small resistance heaters were attached, one at each end of the copper bar. It was found that the smaller the spacing between the copper bar and the inner surface of the specimen the better the temperature was controlled. One thermocouple was embedded in each end of the copper bar. These served as the sensors for two Research Incorporated Thermac Model TC5192 temperature controllers used to control heat in the copper bar.

During the tests, specimen temperature was measured at six positions by means of chromel-alumel thermocouples with fused hot junctions. Two of these thermocouples were cemented on the inner surface of the specimen at positions slightly beyond the gage length. Four thermocouples were attached with spring clips on the outer surface at equal distances within the gage length of the specimen. It was observed that the temperatures could be maintained constant through the test period within $\pm 1/2^{\circ}\text{F}$ except $\pm 1^{\circ}\text{F}$ for tests at 2000 psi. The temperature variation along the gage length was uniform at 75°F and 102°F , but increased to 2°F on the outside and 7°F on the inside of the specimen at 150°F .

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Experimental Procedure and Results

The rate of heating the specimen, both externally and internally, was controlled to about 2°F per minute. After reaching the test temperature the specimen was kept at that temperature for one hour before the loads were applied. Pilot tests indicated that a soaking period from 0.5 hr. to 5.0 hr. at the test temperature (75°F to 160°F) did not show a significant difference in creep response under the same loading conditions. The temperature was maintained at a constant value throughout the testing period. Some adjustments of the set points of the controllers were required, however.

In performing the constant stress creep tests, the specimen was loaded for a one-hour period for each test. After unloading at the end of one hour, the specimen was left to recover at zero stress at the same test temperature for about one hour. After that, heat was gradually turned down until it reached room temperature and the specimen was left to recover at room temperature until the strain had returned to less than 40×10^{-6} in./in. or appeared to have stabilized before proceeding to the next test. Tests 29, 57, 58, 82 and 98 followed tests whose recovery stabilized at 84, 124, 168, 57 (tension) and 114 (tension) micro in./in., respectively. The zero strain for each test was taken to be the strain reading at temperature just prior to loading.

Thermal expansion does not affect the strain in torsion except to alter the gage length slightly. In the present experiments the gage length of the extensometer was fixed at room temperature and no correction was made for the change in gage length with temperature. The thermal expansion was 35.2×10^{-6} in./in./°F. Thus, at 160°F the gage length increased about 0.3 per cent.

The single-step creep test program is outlined in Table I. Strain versus time curves for constant stress are shown in Fig. 1 and 2. Some of the recovery data following the creep tests are shown in Fig. 3. Another type of loading

history reported in this paper was multi-step creep at elevated temperatures. The results of these experiments are shown in Fig. 4, together with the corresponding loading programs.

In previous work [9,10] on the same material at room temperature it was found possible to describe the results of creep strain ϵ_{ij} versus time t under constant stress by a power law with a constant exponent n independent of magnitude or state of combined stress,

$$\epsilon_{ij} = \epsilon_{ij}^0 + \epsilon_{ij}^+ t^n, \quad (1)$$

where ϵ_{ij}^0 , ϵ_{ij}^+ are functions of stress but independent of time. For torsion subscripts ij become 12 . From the results shown in Fig. 1-4, it seems that (1) can also be used to describe the strain versus time relationship of the torsion creep tests at elevated temperature. To examine the effect of temperature a best fit of each data set to (1) was calculated using a least squares method. The results showed the exponent n to be essentially independent of stress and temperature. There was considerable scatter in n but no definite trend as a function of either stress or temperature. The values of ϵ_{12}^0 showed scatter and some temperature effect; ϵ_{12}^0/τ increased from about 0.26 to 0.30 from 75°F to 160°F. On the other hand, ϵ_{12}^+ showed a strong effect (more than doubled over the same temperature range) but showed considerable scatter. The difference between the effect of temperature on ϵ_{12}^0 and ϵ_{12}^+ is in accord with thermal considerations. The effect of temperature on the time-independent strain (stiffness) is small because it results essentially from altering the atomic spacing and hence the bond forces. The effect of temperature on the time-dependent strain, however, is large because of the change in energy of the atomic vibrations and hence the change in frequency with which interchange of atomic bonds can take place.

In view of the scatter in n resulting from the individual determinations, the value used for room temperature tests of this material in earlier work [8-10] $n = 0.143$ was selected as a common value for all stresses and temperatures. Also in view of the apparent small effect of temperature on ϵ_{12}^0 (relative to the effect on ϵ_{12}^+), this parameter was taken to be independent of temperature. Using these restrictions, values of ϵ_{12}^0 and ϵ_{12}^+ were recalculated and are shown in Table I. In the analysis which follows in a later section, (1) with the values of ϵ_{12}^0 , ϵ_{12}^+ and n shown in Table I were employed rather than the experimental data. Other choices were also tried, including other values of n and taking ϵ_{12}^0 to be a linear function of temperature, but the overall agreement with the test data was not as good as for the values given in Table I.

Discussion of Results

Experiments of the type described are expensive and time consuming--especially if a new specimen must be employed for each experiment. Also, if many specimens are used variation in the material will affect the results. Thus, it was decided to follow the practice in similar work by the authors at 75°F of using only one specimen and following each creep experiment by a recovery period to allow the specimen to resume its original condition before proceeding to the next experiment. Since the effect of temperature in combination with stress on the structural stability of the material was not known, the experiments progressed in an exploratory manner in which the severity of the conditions of temperature and stress were increased periodically.

To minimize possible changes in the material resulting from extended time under stress at elevated temperature, the duration of time at temperature for each test was limited by allowing only one hour at temperature following

removal of stress plus one to several days at 75°F for recovery to occur. This was a compromise, since it is clear that recovery requires a much longer time at temperature than the duration of the preceding creep.

The program of experiments, completed in 21 months, included 118 creep tests under torsion, tension and combined tension and torsion at temperatures ranging from 75°F to 160°F and stresses up to 4000 and 2000 psi in tension and torsion, respectively. Now and then an experiment was repeated to furnish evidence as to how much the creep behavior was changing as a result of the history of stress and temperature. These repeated tests for torsion are summarized in Table II in which the strain after one hour under load is given for each testing condition. An example of the repeatability of the shape of the creep curves is shown by tests 22 and 62 in Fig. 1. Note that 40 creep tests were made on this specimen between these two tests.

Examination of all the data showed an extensive middle period in which there was essentially no change in material response. In tension (not shown in Table II) there was some change in material response--especially toward the end of the program. However, there were larger changes in torsion. It was observed that the torsional strain after one hour of creep at a given stress increased rather continuously as the test program progressed up to about test 20. Fifteen of these were at temperatures of 75°F to 102°F; the other 5 were at higher temperatures up to 145°F. After this adjustment period the material response appeared to remain nearly constant until about the period that the highest stresses were applied, which started with test 78. Analysis of the data showed clearly that the values of ϵ_{12}^0 and n were not affected by the history of stress and temperature; the observed changes in response occurred in the time-dependent coefficient ϵ_{12}^+ .

It was also observed that the residual strains as measured just before heating for the next experiment tended to change with time or number of tests. They increased substantially in torsion and occasionally in tension, but generally they tended to decrease in tension. The larger changes seemed to correlate with increases in test temperature. This may be explained by the short recovery times at temperature causing some strain to be locked in when the temperature was lowered.

The specimen was annealed a few times, as follows: after test 15: 2 hr. at 135°F; after test 59: 1 hr. at 160°F; after test 82: 15 hr. at 135°F; after test 99: 2 hr. at 160°F. It was not clear that these had any large effect. No significant changes in residual strains occurred in the first 12 tests at temperatures up to 115°F. However, a step increase occurred in torsion after test 13 at 131°F. Another large increase occurred after test 18 at 145°F.

Another manifestation of residual stresses was that on changing from torsion tests at elevated temperature to tension creep tests at elevated temperature it was observed that the specimen began to recover in torsion when the temperature was raised and continued to recover during the tension creep test. In tests 45, 57, 58 and 64 significant recovery in torsion occurred during the one-hour period at temperature prior to loading. If this recovery affected the subsequent creep, there was no evidence of it in the test data.

The fact that the effect of the sequence of tests on the creep behavior was greater in torsion than tension may result from the fact that the strain distribution is not uniform in torsion whereas it is in tension. Even in the thin-walled tube employed there is a variation of about 12 per cent in stress and in strain for torsion between the inside and outside surfaces of the specimens. The nonuniformity of stress and strain in torsion was taken into account

in the analysis by employing the average stress and strain (the values at the mid-thickness of the tube).

One of the last creep tests performed was test 110, Fig. 2. This test employed the same high stress, 2 ksi, the same high temperature, 135°F, but the opposite sense of torque to that of test 82. The results are very similar for the two tests except that the creep rate was greater in test 110 than test 82. The agreement seems good especially in view of the fact that 25 severe tests were performed between the two.

Comparing the results in Table I at 75°F with the results of similar tests [12] of a different specimen of the same material shows excellent agreement for ϵ_{12}^0 except for test 85 whose value is somewhat larger. The values of ϵ_{12}^+ in Table I for 75°F are somewhat larger than reported in [12] for pure torsion. They agree more closely with data in [12] for combined tension and torsion in which the tension was between 2000 to 3000 psi except for test 85 which was nearer to the value for tension of 4000 psi. Thus the present results may be influenced by synergistic effects resulting from combining the applied torsion stresses with residual tension strains locked in from prior experiments.

In the analysis of the effect of temperature on creep behavior in this paper on torsion creep and in a companion paper on tension creep using data from the same test series, the first twenty experiments have been ignored because of the rapid changes in creep response which seemed to be occurring. The experiments employed in the analysis were taken from the period when apparent changes were small. They were influenced to some extent by the sequence of tests and the residual strains locked in, however. These effects, which may have tended to accentuate the observed nonlinearity, were ignored in the analysis.

Analysis

Nonlinear Constitutive Equation

A nonlinear constitutive equation in the multiple integral form has been derived by several investigators. Under a pure shear (torsion) stress state, the following equation can be obtained (see for example [5]):

$$\begin{aligned} \epsilon_{12}(t) = & \int_0^t G_1(t-\xi_1, T) \dot{\tau}(\xi_1) d\xi_1 \\ & + \int_0^t \int_0^t \int_0^t G_3(t-\xi_1, t-\xi_2, t-\xi_3, T) \dot{\tau}(\xi_1) \dot{\tau}(\xi_2) \dot{\tau}(\xi_3) d\xi_1 d\xi_2 d\xi_3, \quad (2) \end{aligned}$$

where ϵ_{12} is the torsional strain in tensor notation*, τ is the torsional stress, $\dot{\tau}(\xi) = \partial \tau(\xi) / \partial \xi$. The fact that the torsional strain in (2) depends only on the odd functions of torsional stress implies that reversing the sense of the torsional stress results only in reversing the sense of the torsional strain, the magnitude of the torsional strain remaining unchanged. This is an essential requirement for material isotropy and objectivity. The kernel functions G_1 and G_3 of (2) depend on temperature, time and material. Under constant stress σ , $\sigma(t) = \sigma H(t)$, where $H(t)$ has the value 1 when $t \geq 0$ and 0 when $t < 0$. Thus under isothermal conditions and constant stress (2) becomes

$$\epsilon_{12}(t, T) = G_1(t, T)\tau + G_3(t, T)\tau^3. \quad (3)$$

* $\epsilon_{12} = \gamma/2$ where γ is the engineering shear strain.

Determination of Kernel Functions

A comparison of (3) with (1) suggests that one possible form for the kernel functions G_1 and G_3 is:

$$G_1(t, T) = \alpha_1 + \beta_1(T)t^n, \quad (4)$$

$$G_3(t, T) = \alpha_3 + \beta_3(T)t^n.$$

Inserting (4) into (3) and comparing with (1) results in

$$\epsilon_{12}^0(\tau) = \alpha_1\tau + \alpha_3\tau^3, \quad (5a)$$

$$\epsilon_{12}^+(\tau, T) = \beta_1(T)\tau + \beta_3(T)\tau^3. \quad (5b)$$

In (4) and (5) the coefficients α_1 , α_3 are taken to be independent of temperature, since the test data showed that ϵ_{12}^0 could be taken to be independent of temperature. In the following, the results shown in Table I were used to determine α_1 , α_3 and $\beta_1(T)$, $\beta_3(T)$.

Constants α_1 and α_3 were determined graphically by plotting ϵ_{12}^0/τ versus τ . From (5a)

$$\epsilon_{12}^0/\tau = \alpha_1 + \alpha_3\tau^2. \quad (6)$$

α_1 and α_3 were chosen to yield the best agreement between the parabolic relationship of (6) and the test results. See Fig. 5.

The method of determination of $\beta_1(T)$ and $\beta_3(T)$ was as follows. The time-dependent part of the strain ϵ_{12}^+ from Table I versus temperature T was plotted for each stress level, as shown in Fig. 6. After that ϵ_{12}^+/τ versus τ for selected temperature levels (75°F, 100°F, 120°F and 140°F) were plotted as shown in Fig. 7. In making these plots, ϵ_{12}^0 at the temperatures 100°F,

120°F, 140°F at all stress levels were obtained from the lines of Fig. 6. In another paper [15] a similar plot of the time-dependent part of tensile strain divided by the tensile stress ϵ_{11}^+/σ versus σ for different temperatures showed that the plots for different temperatures had the same shape. They deviated only by a vertical shift. However, the plots shown on Fig. 7 have different shapes. This difference between tension and torsion may result from locked-in residual stresses, as discussed in a prior section.

From (5b)

$$\frac{\epsilon_{12}^+(\tau, T_1)}{\tau} = \beta_1(T_1) + \beta_3(T_1)\tau^2, \quad (7)$$

where $T_1 = T - 75^\circ\text{F}$. 75°F was chosen as the reference temperature because it was the lowest test temperature in this investigation. The plots of Fig. 7 show that both β_1 and β_3 are temperature dependent. β_1 increases with increase of temperature, while the effect of temperature on β_3 is the reverse. $\beta_1(T_1)$ and $\beta_3(T_1)$ were determined as follows. At each selected temperature β_1 and β_3 were chosen to yield the best fit of the parabolic relationship (7) to the data points as shown in Fig. 7. The effect of temperature on β_1 and β_3 was determined by plotting β_1 versus T_1 and β_3 versus T_1 , as shown in Fig. 8. The resulting curves were represented by

$$\beta_1(T_1) = \theta_1 + \theta_2 T_1 + \theta_3 T_1^2, \quad (8a)$$

$$\beta_3(T_1) = \theta_4 + \theta_5 T_1, \quad (8b)$$

where $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ have the values shown in Fig. 8.

The kernel functions determined in this section are summarized as follows

$$\epsilon_{12}(\tau, T, t) = (\alpha_1 + \alpha_3 \tau^2) \tau + [(\theta_1 + \theta_2 T_1 + \theta_3 T_1^2) + (\theta_4 + \theta_5 T_1) \tau^2] \tau t^n \quad (9a)$$

$$= (0.259 + 0.0065 \tau^2) \tau + [0.025 + 0.00074 T_1 + 0.000007 T_1^2 + (0.004 - 0.00011 T_1) \tau^2] \tau t^{0.143}, \quad (9b)$$

where $T_1 = T - 75^\circ\text{F}$, ϵ_{12} is in per cent, and τ is in ksi (1 ksi = 1000 pounds per sq. in.). Predictions of creep strains using (9) were made for each stress and temperature and are shown by the lines in Fig. 1 and 2.

A comparison between a linear and a nonlinear representation may be made by observing the plus symbols at 1 hr. in Fig. 1 and 2. These represent the strains at 1 hr. computed from the linear terms of (9b) by taking $\alpha_3 = \theta_3 = \theta_4 = \theta_5 = 0$. The designations adjacent to the symbols (160L for example) indicate values computed from linear theory at the indicated temperature. Figure 2 shows that the nonlinear terms have a considerable effect.

Analysis of Creep Resulting from Multi-step Loading

In principle, (2) is able to describe creep behavior under various time-dependent loading histories provided the kernel function $G_3(t - \xi_1, t - \xi_2, t - \xi_3)$ is known for the different time arguments $(t - \xi_1)$, $(t - \xi_2)$, $(t - \xi_3)$. In the previous section, G_3 was determined only for $(t - \xi_1) = (t - \xi_2) = (t - \xi_3)$. To completely determine G_3 requires a large number of multi-step creep tests such as described in [7]. This is very time consuming and does not yield the desired accuracy. Therefore, several approximate methods have been proposed to reduce the constitutive equation to a single integral form [16,17,8,18,13]. Among them, the modified superposition method [17,13] has proven to be one of the simplest and yet yields satisfactory

predictions. Hence, only the "modified superposition principle" is employed in this section to predict creep under multiple steps of loading.

Modified Superposition Method

The modified superposition method [17] suggested that for a stepwise change of stress from τ_{N-1} to τ_N at time $t = t_N$, the corresponding creep strain at $t > t_N$ can be represented by the following form

$$\epsilon_{12}(t) = \sum_{i=0}^N [f(\tau_i, t-t_i) - f(\tau_{i-1}, t-t_i)] \quad t > t_N, \quad (10)$$

where

$$\epsilon_{12}(t) = f(\tau, t) = G_1(t)\tau + G_3(t)\tau^3 \quad (10a)$$

is the nonlinear creep function for constant stress as given by (3) and (9). For an arbitrary varying stress history, which can be considered as a limiting case of an infinite number of infinitesimal steps of stress, (10) becomes

$$\epsilon_{12}(t) = \int_0^t \frac{\partial f[\tau(\xi), t-\xi]}{\partial \tau(\xi)} \frac{d\tau(\xi)}{d\xi} d\xi, \quad (11)$$

where the stress history $\tau(\xi)$ is considered to be differentiable.

Multi-step loading histories such as those shown in Fig. 3 and 4 can be expressed in one equation as follows

$$\tau(t) = \tau_0 H(t) + (\tau_1 - \tau_0) H(t-t_1) + (\tau_2 - \tau_1) H(t-t_2) \quad (12)$$

where τ_i is the stress at each step. $H(t-t_i)$ is the Heaviside unit function.

It has the value 1 when $t \geq t_i$, 0 when $t < t_i$. Inserting (12) into (11) yields the following results for each loading period at constant temperature:

$$(1) \quad \epsilon_{12}(t) = f(\tau_0, t), \quad t < t_1 \quad (13a)$$

$$(2) \quad \epsilon_{12}(t) = f(\tau_0, t) - f(\tau_0, t-t_1) + f(\tau_1, t-t_1), \quad t_1 \leq t < t_2 \quad (13b)$$

$$(3) \quad \epsilon_{12}(t) = f(\tau_0, t) - f(\tau_0, t-t_1) + f(\tau_1, t-t_1) - f(\tau_1, t-t_2) + f(\tau_2, t-t_2), \quad t > t_2 \quad (13c)$$

etc., where $f(\tau, t)$ is given by (9b) for the present purposes.

Recovery Following Creep at Constant Stress

For recovery following creep for 1 hr. at constant stress τ_1 let $\tau_2 = 0$ and $t_1 = 1.0$ in (12). The recovery strain can be represented by (13b) as follows

$$\epsilon_{12}(t) = \epsilon_{12}^0(\tau_1) + \epsilon_{12}^+(\tau_1)t^n - [\epsilon_{12}^0(\tau_1) + \epsilon_{12}^+(\tau_1)(t-1.0)^n] \quad , \quad (14)$$

or

$$\epsilon_{12}(t) = \epsilon_{12}^+(\tau_1)[t^n - (t-1.0)^n] \quad ,$$

where $\epsilon_{12}^0(\tau)$, $\epsilon_{12}^+(\tau)$ are given by (5a), (5b). The recovery strains following constant stress creep were predicted by (14) from the creep data described by (9b), as shown by Fig. 3. The agreement with the actual recovery data is excellent, except at 160°F. The recovery strain predicted by (14) indicates that after a long time of recovery, $t \gg 1.0$, the recovery strain would approach zero, as observed, except at 160°F.

Prediction of Multi-step Creep Behavior at Different Constant Temperatures

For the test programs shown in Fig. 4, (13a), (13b) and (13c) were used to predict the creep strain following each change in stress. The predicted strains were calculated from the constants shown in (9b), which were determined from the single-step constant-load tests shown in Fig. 1 and 2. It should be noted that after partial unloading the strain decreases for a period and then begins to increase; see Fig. 4b.

Conclusions

The following conclusions can be drawn from the results of the present investigation of polyurethane at temperatures from 75°F to 160°F.

Results of the constant torsional stress creep tests at different temperatures indicate that creep strains are separable into time-independent and time-dependent parts. The time dependence was found to be adequately described by a power function of time whose exponent was independent of stress and temperature. The coefficient of the time-dependent strain was found to be a strongly nonlinear function of stress and strongly dependent on temperature. On the other hand, the time-independent term was a weakly nonlinear function of stress and slightly dependent on temperature (the best overall representation considered the time-independent term not to be a function of temperature). The temperature affected both the linear stress term and the nonlinear stress term of the time-dependent term. Furthermore, the effect of temperature on the nonlinear stress term was negative; an increase in temperature resulted in a decrease of the coefficient of the third order stress term, though the total time-dependent part of the strain increased with an increase of temperature.

The modified superposition method was found to provide a good prediction of the creep behavior under multi-step loadings and recovery at different constant temperatures.

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Table I
Torsion Creep Experiments

Test	Shearing Stress, ksi	Temperature, °F	Experimental Results, ϵ_{12}
24	0.5	75	$0.13 + 0.014t^n$
26	"	102	$0.13 + 0.024t^n$
27	"	115	$0.13 + 0.030t^n$
29	"	131	$0.13 + 0.041t^n$
32	"	145	$0.13 + 0.057t^n$
57	"	160	$0.13 + 0.068t^n$
22	1	75	$0.265 + 0.032t^n$
64	"	102	$0.265 + 0.056t^n$
66	"	115	$0.265 + 0.070t^n$
68	"	126	$0.265 + 0.081t^n$
30	"	131	$0.265 + 0.093t^n$
73	"	135	$0.265 + 0.082t^n$
58	"	160	$0.265 + 0.137t^n$
23	1.5	75	$0.409 + 0.042t^n$
25	"	102	$0.409 + 0.071t^n$
28	"	115	$0.409 + 0.090t^n$
55	"	131	$0.409 + 0.118t^n$
45	"	135	$0.409 + 0.135t^n$
103	"	145	$0.409 + 0.152t^n$
85	2	75	$0.570 + 0.090t^n$
93	"	102	$0.570 + 0.116t^n$
94	"	115	$0.570 + 0.128t^n$
82	"	135	$0.570 + 0.180t^n$
98	"	145	$0.570 + 0.190t^n$

$n = 0.143$ for all tests.

Table II
Repeated Experiments in Torsion

Test	Stress, ksi	Temperature, °F	Strain at 1 hr., per cent	Test	Stress, ksi	Temperature, °F	Strain at 1 hr., per cent
12	0.5	75	0.1390	31	1	126	0.3410
17	"	"	0.1444	68	"	"	0.3469
24	"	"	0.1443 ⁺	13	"	131	0.3592
5	"	102	0.1447	30	"	"	0.3591
26	"	"	0.1532 ⁺	73	"	135	0.3471
8	"	115	0.1552	74	"	"	0.3448
27	"	"	0.1592 ⁺	9	1.5	75	0.3460
1	1	75	0.2777	23	"	"	0.4507 ⁺
10	"	"	0.2861	45	"	135	0.5451 ⁺
15	"	"	0.2933	59	"	"	0.5244
16	"	"	0.2965	56	"	145	0.5342 [*]
19	"	"	0.3024	103	"	"	0.5450 [*]
22	"	"	0.2978 ⁺	3	2	75	0.5790
62	"	"	0.2988	11	"	"	0.5926
6	"	102	0.3020	34	"	"	0.6060
14	"	"	0.3122	36	"	"	0.6153
20	"	"	0.3197	85	"	"	0.6604 ⁺
64	"	"	0.3216 ⁺	44	"	102	0.6421
7	"	115	0.3222	93	"	"	0.6858 ⁺
21	"	"	0.3319	82	"	135	0.7497
61	"	"	0.3458	110	-2 ^{**}	"	0.7539
66	"	"	0.3358 ⁺				

* Strain at 1/2 hr.

** Torque in opposite sense to all other tests.

⁺ Tests used in Fig. 1 and 2.

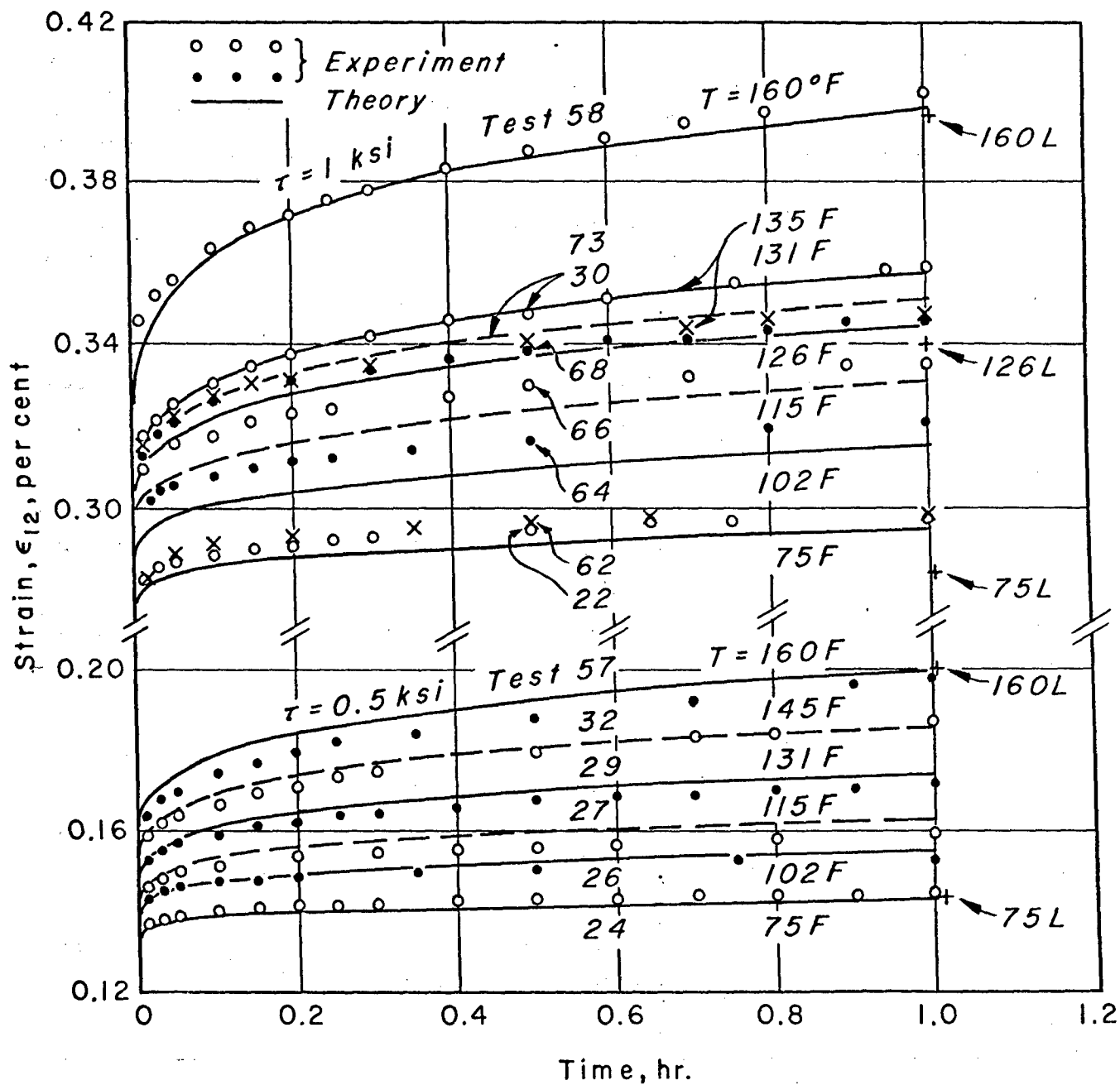


Fig. 1. Creep Curves for Shearing Strains

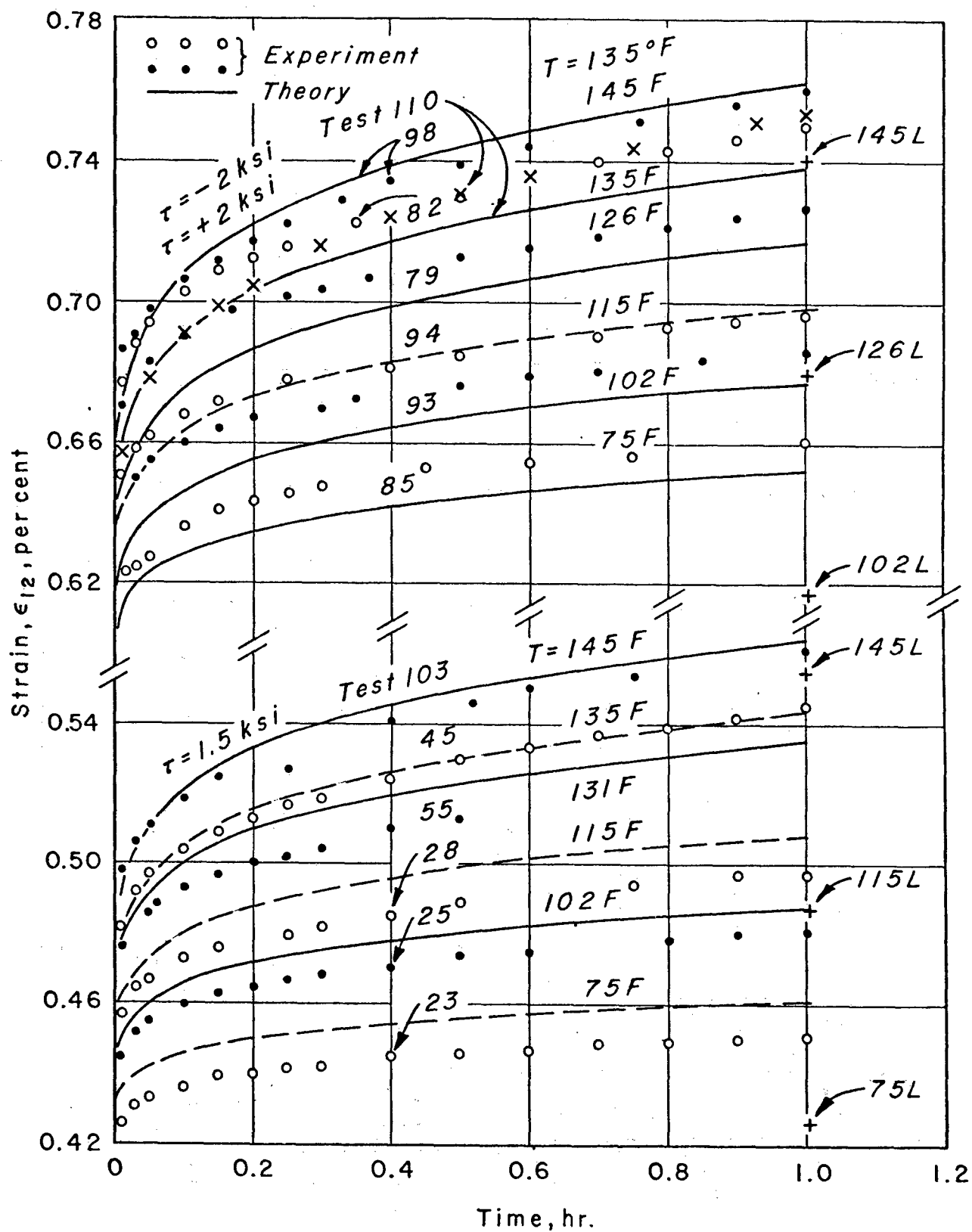


Fig. 2. Creep Curves for Shearing Strains

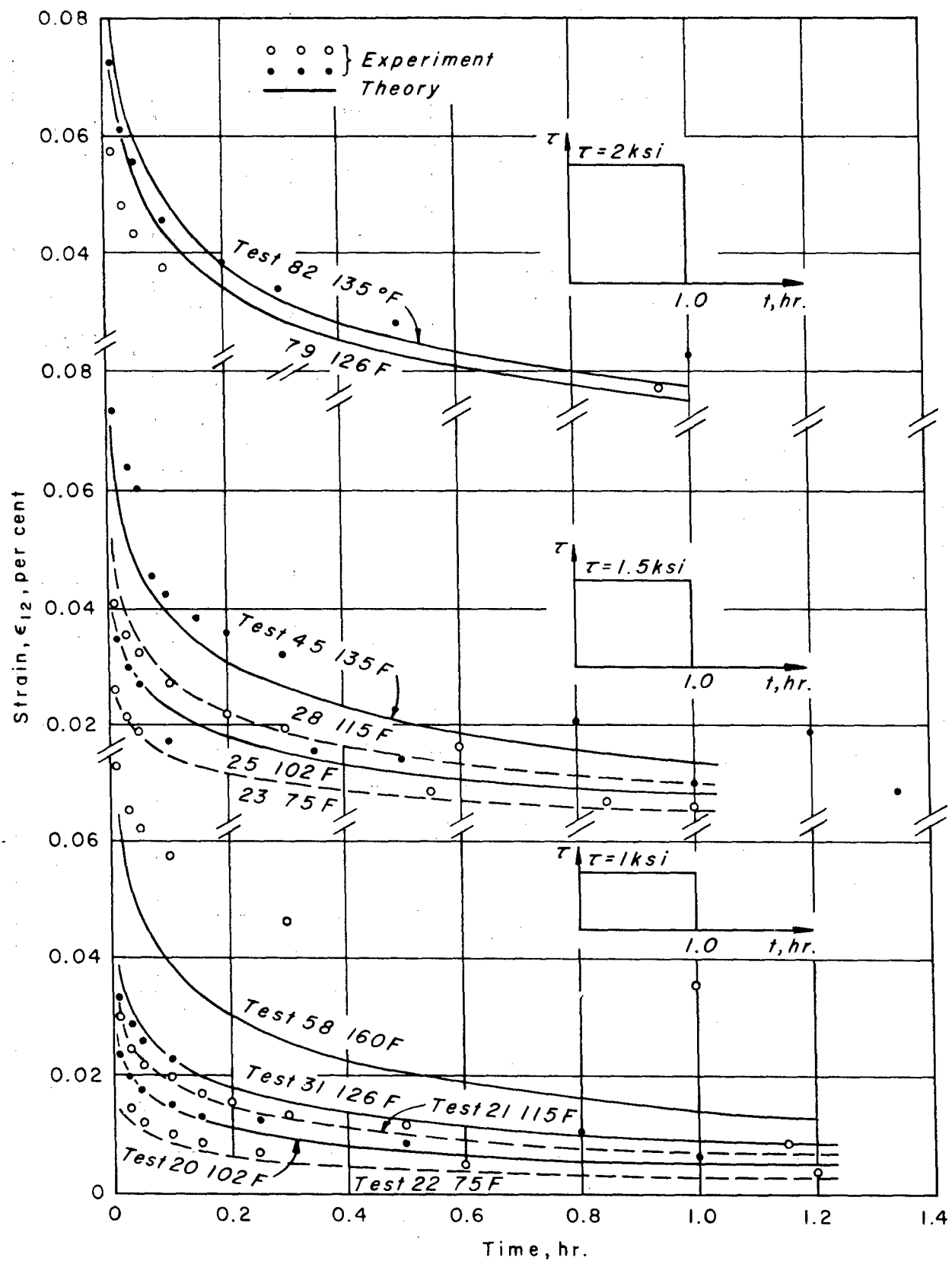


Fig. 3. Recovery Curves

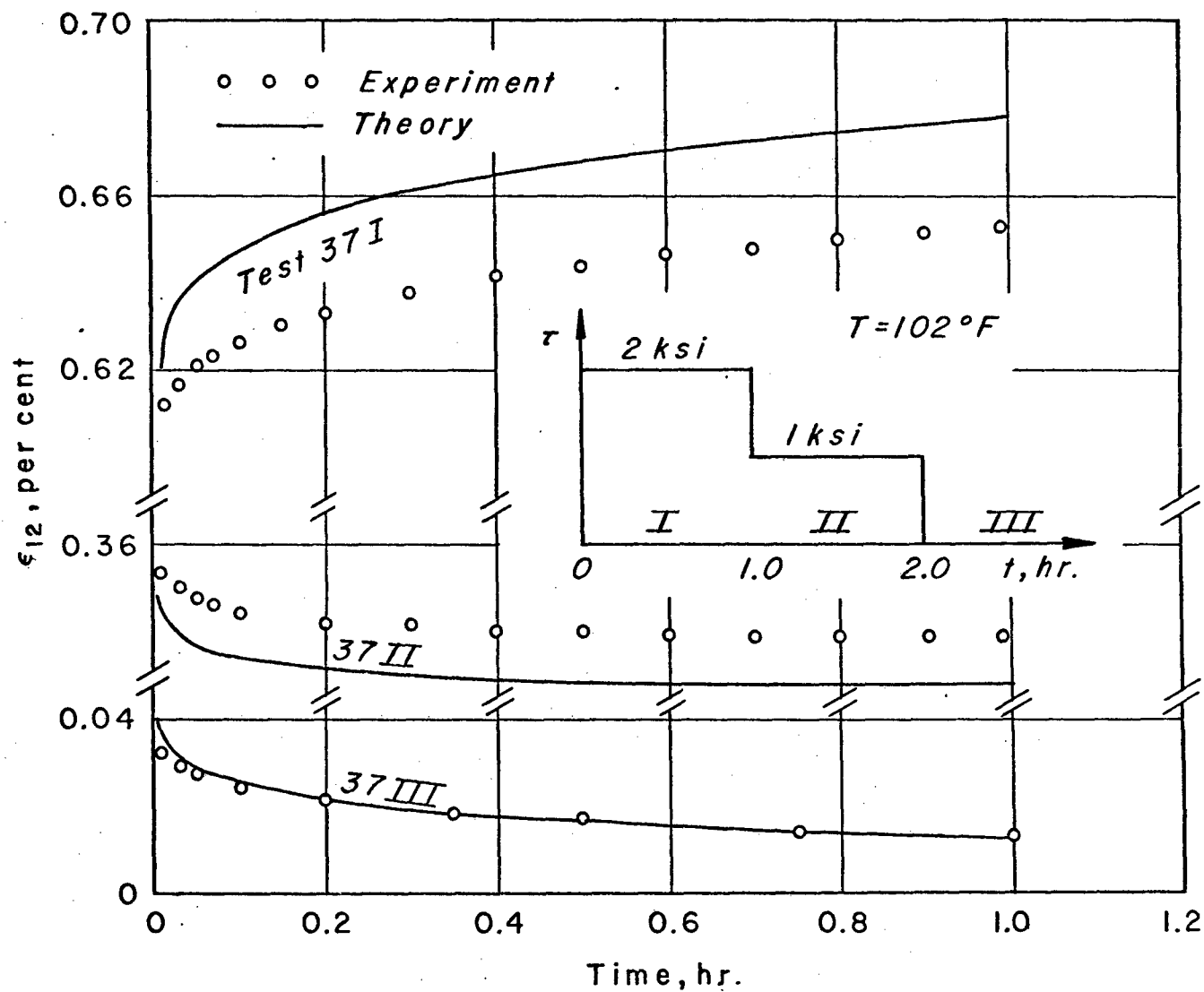


Fig. 4a. Step-loading Creep and Recovery

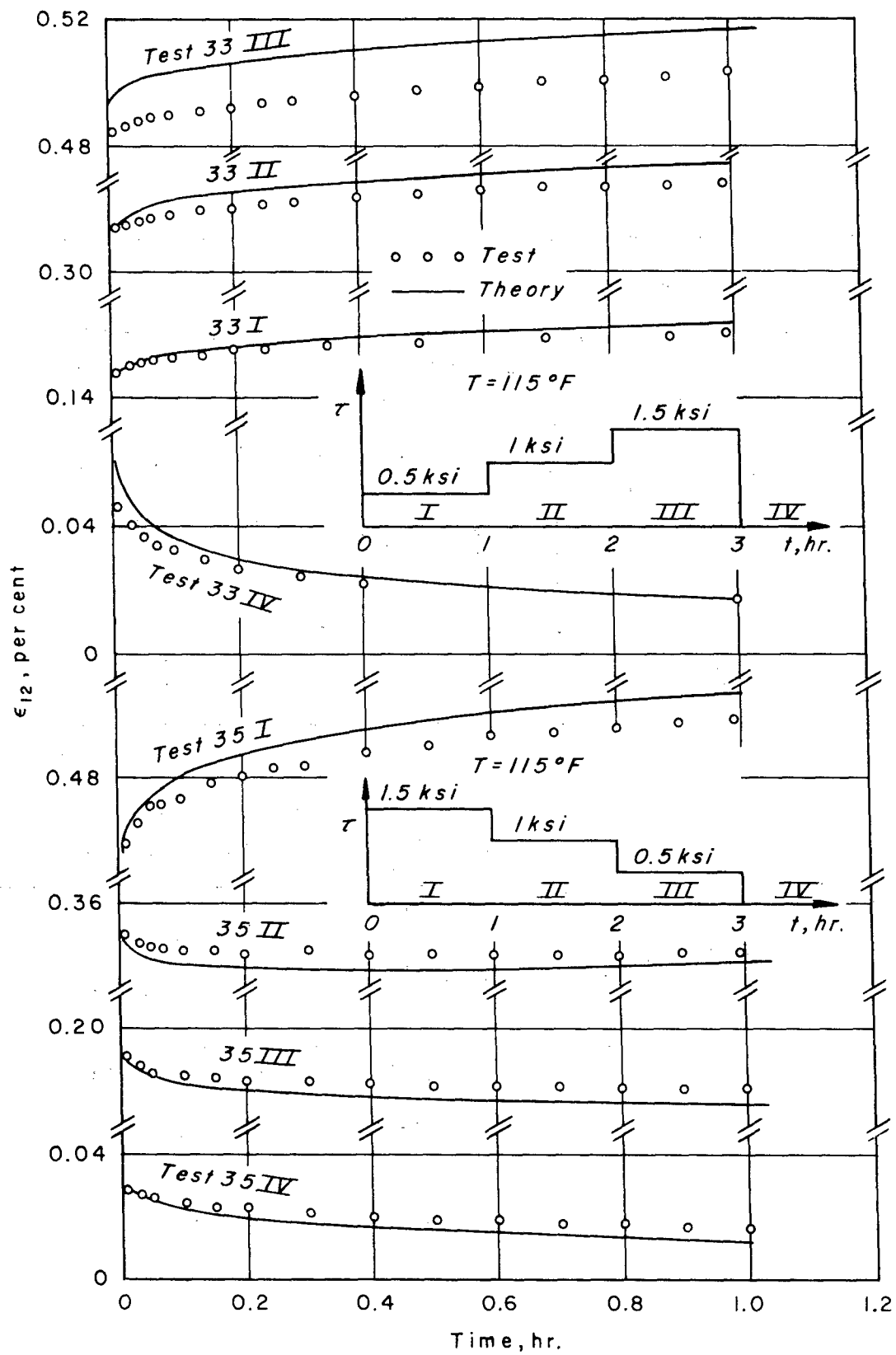


Fig. 4b. Step-loading Creep and Recovery

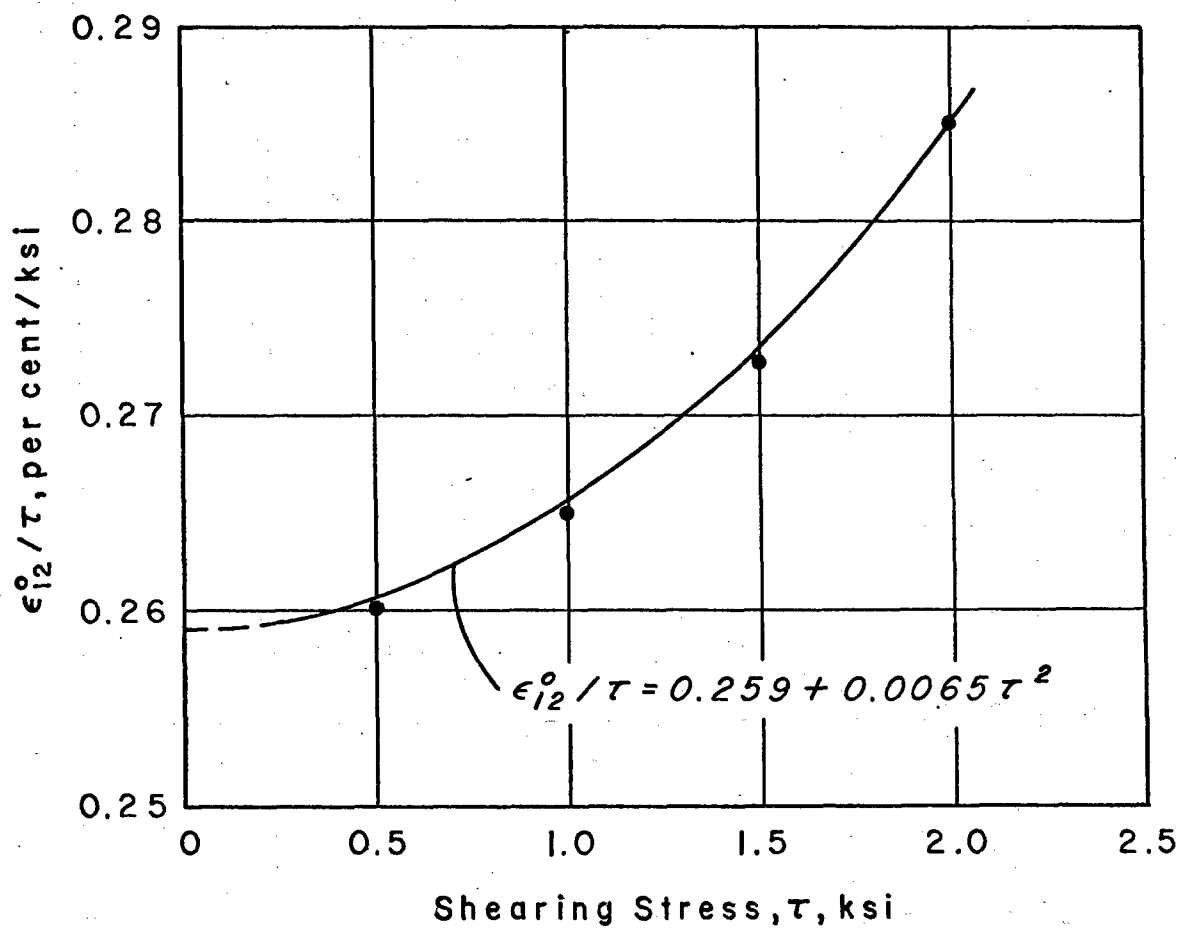


Fig. 5. Effect of Stress on Time-independent Strain

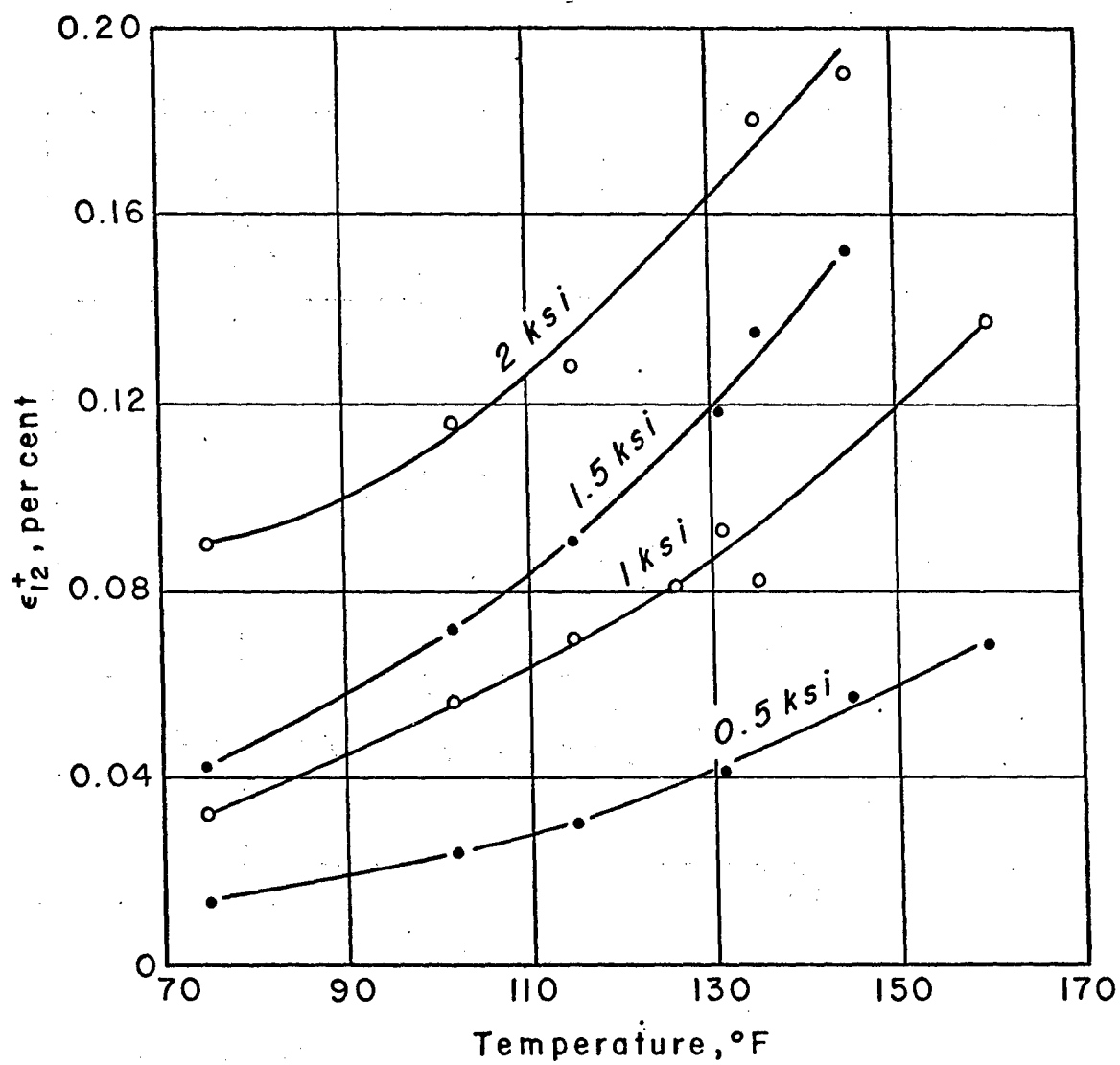


Fig. 6. Effect of Temperature on Time-dependent Coefficient

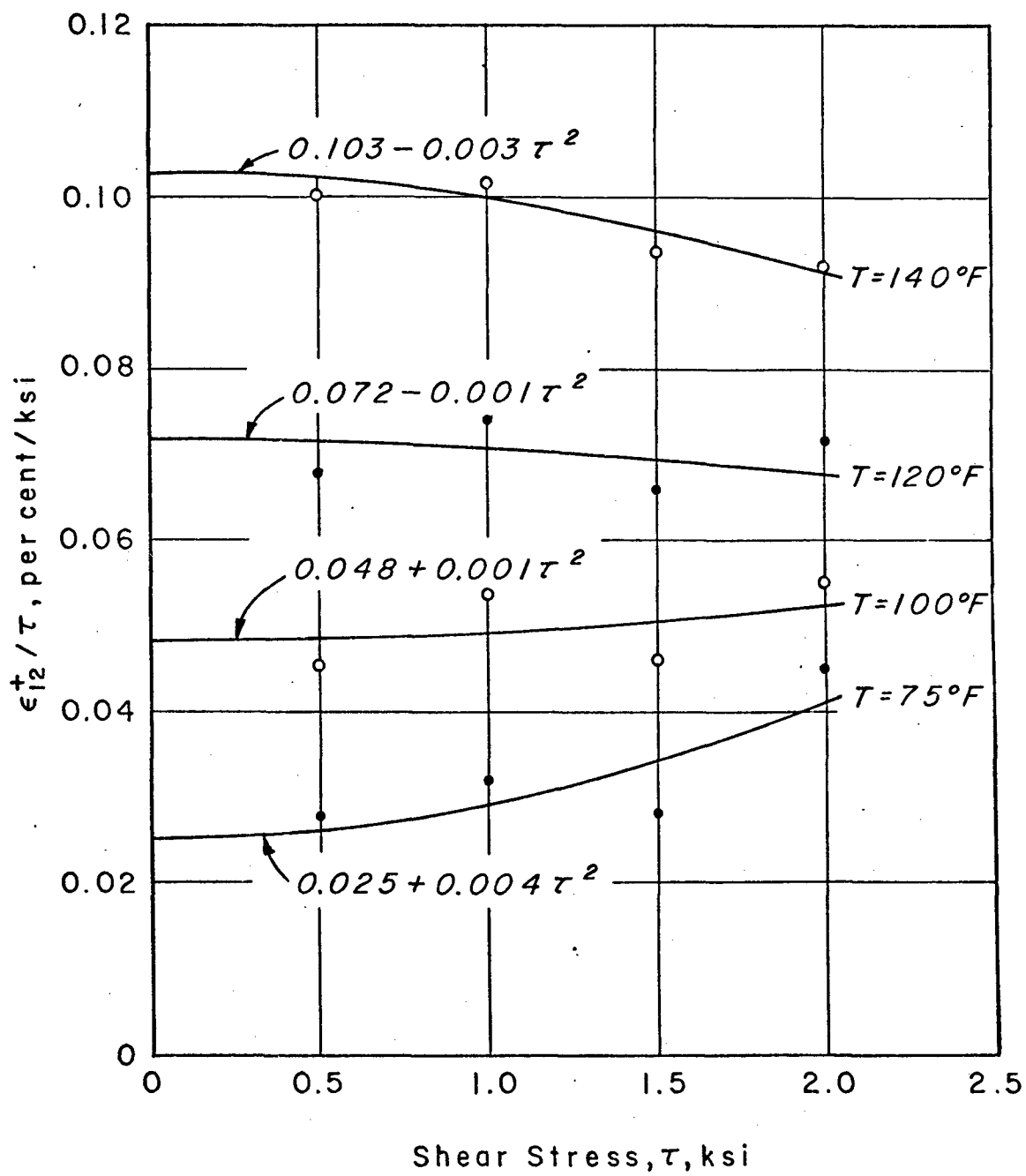


Fig. 7. Effect of Stress on Time-dependent Coefficient

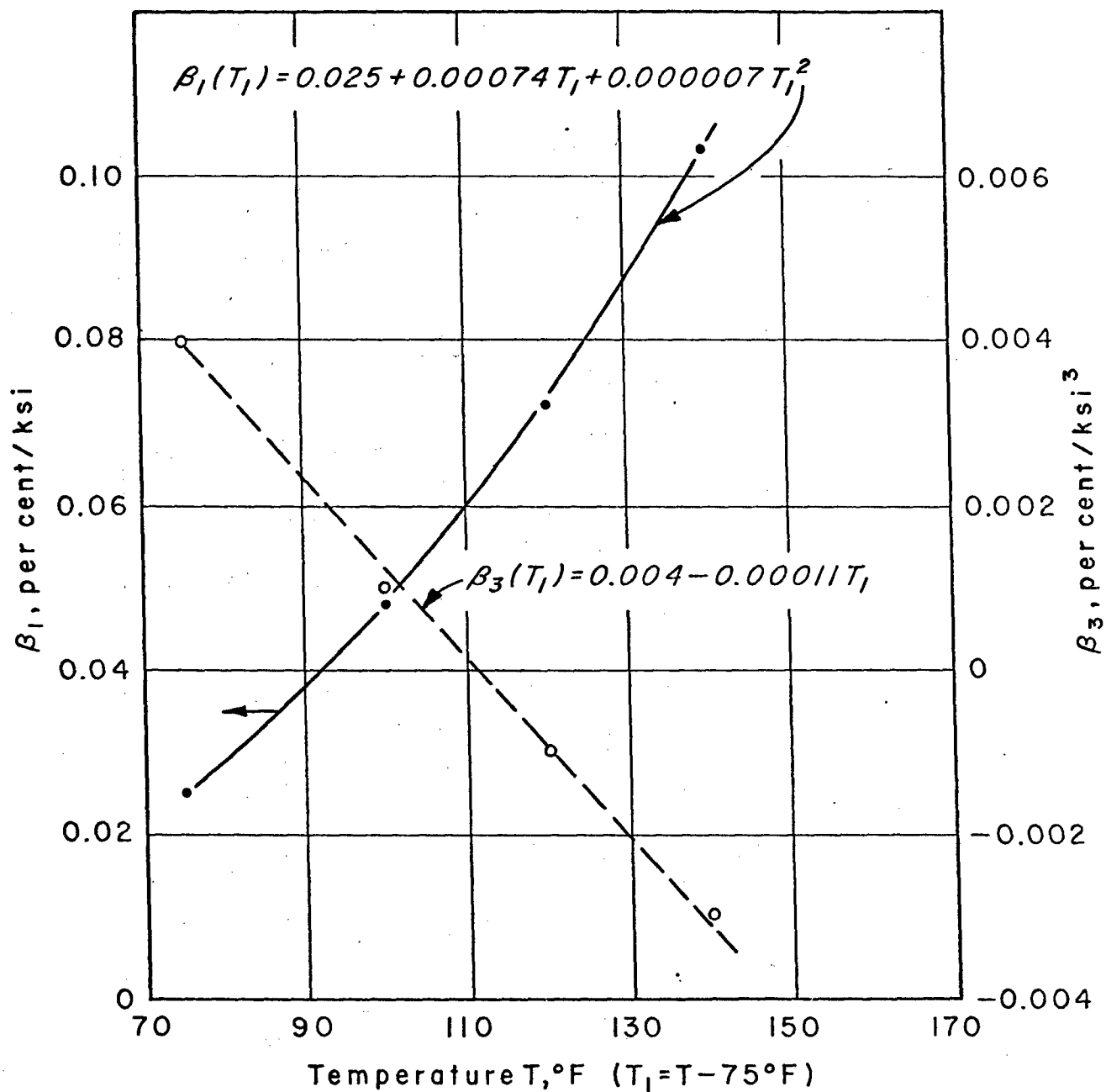


Fig. 8. Temperature Dependence of Parameters β_1 , β_3 in Eq. 7